A Dynamic Model of International Fishing

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Abstract: International commercial fishing is examined when the harvesting countries form a coalition, and at each time period the total profit of the coalition is maximized. The fishing activity is combined with population dynamics to derive a special dynamic systems model for the fish stock. A complete description is presented for the number of positive equilibria and for the monotonic and asymptotical properties of the fish stock. The effect of the entry of a new nation into the coalition is also examined. The harvesting rates of cooperating and competing countries are compared and it is shown that the fish stock is more stable in the case of cooperation.

Keywords: International commercial fishing; coalition; population dynamics; fish stock

1. INTRODUCTION

In his earlier paper Okuguchi [1998] has analyzed commercial fishing under imperfect competition when two countries harvest a single species in an open sea area. These results have been extended by Szidarovszky and Okuguchi [1998] for the general case of *N* countries when each harvesting country is able to sell fish in her own and all other's markets. It is also assumed that at each time period a Cournot equilibrium is formed and the harvested amount of fish is always equal to the equilibrium amount.

In this paper, a modified model will be examined in which we assume that at each time period the harvesting countries form a coalition and their total profit is maximized. The resulting dynamic model is different than the one earlier examined by Szidarovszky and Okuguchi [1998]. The monotonic and asymptotical behavior of the trajectory of this modified system will be investigated, the effect of the entry of a new nation into the coalition will be examined, and the harvesting rates of the two cases will be compared. We will show that cooperation among the harvesting countries makes the fish stock more stable in the long run.

2. THE BASIC MODEL

Consider an open sea area where n countries harvest a single species. As in Szidarovszky and Okuguchi [1998] let x_{lj} denote the amount of fish harvested by country l and sold in country j ($l = 1,2,\cdots,n;\ j = 1,2,\cdots,n$). The price of fish in country j is assumed to be the following:

$$P_j = a_j - b_j Y_j \tag{1}$$

with $Y_j = x_{1j} + x_{2j} + \cdots + x_{nj}$. The fishing cost of country l is given by:

$$C_l = c_l + \gamma_l X_l^2 / X, \tag{2}$$

where $X_l = x_{l1} + x_{l2} + \cdots + x_{ln}$, and X is the total level of fish stock. This special form of the cost function can be derived from the production function of the fishing activity as shown in Szidarovszky and Okuguchi [1998]. Here c_l and γ_l are positive parameters. The profit of country l is therefore:

$$\Pi_l = \sum_{j=1}^{n} P_j x_{lj} - (c_l + \gamma_l X_l^2 / X). \tag{3}$$

Assume that the fish harvesting countries form a grand-coalition. The profit of the coalition is obtained as:

$$\Pi = \sum_{l=1}^{n} \Pi_l = \sum_{j=1}^{n} (a_j - b_j Y_j) Y_j$$

$$- \sum_{l=1}^{n} (c_l + \gamma_l X_l^2 / X), \tag{4}$$

which is concave in the variables x_{lj}. Assuming an interior optimum, the first order conditions can be written as the following linear equations:

$$\frac{\partial \Pi}{\partial x_{lj}} = -b_j Y_j + (a_j - b_j Y_j)$$

$$- 2\gamma_l X_l / X = 0 \tag{5}$$

for all l and j. This equation can be rewritten as

$$a_j - 2b_j Y_j = 2\gamma_l X_l / X \tag{6}$$

(for all l and j)

showing that

$$a_j - 2b_j Y_j = 2\gamma_l X_l / X = \delta \tag{7}$$

with some constant δ. This implies that in each country the marginal revenue is identical and equal to the identical marginal fishing cost. From equation (7) we have

$$Y_j = \frac{a_j - \delta}{2b_j} \text{ and } X_l = \frac{\delta X}{2\gamma_l} \tag{8}$$

Since $S = \sum_j Y_j = \sum_l X_l$ gives the total harvest rate, from relations (8) a single equation is obtained for δ:

$$\sum_j \frac{a_j}{2b_j} - \delta \sum_j \frac{1}{2b_j} = \delta X \sum_l \frac{1}{2\gamma_l} \tag{9}$$

implying that

$$\delta = \frac{\sum_j \frac{a_j}{b_j}}{X \sum_l \frac{1}{\gamma_l} + \sum_j \frac{1}{b_j}} \tag{10}$$

Hence the total harvest rate is as follows:

$$S = \sum_l X_l = \frac{X \sum_j \frac{a_j}{2b_j} \sum_l \frac{1}{\gamma_l}}{X \sum_l \frac{1}{\gamma_l} + \sum_j \frac{1}{b_j}} \tag{11}$$

We assume that in the absence of fishing the fish stock changes according to the biological law

$$\dot{X} = X(\alpha - \beta X) \tag{12}$$

[see for example Clark, 1976] , therefore it changes according to

$$\dot{X} = X(\alpha - \beta X - \frac{AC}{CX + B}) \tag{13}$$

in the presence of commercial fishing with

$$A = \sum_j \frac{a_j}{2b_j}, \quad B = \sum_j \frac{1}{b_j} \text{ and } C = \sum_l \frac{1}{\gamma_l}$$

In the next section the number of equilibria of system (13) will be determined and the monotonic and asymptotical behavior of the system will be discussed as $t \to \infty$.

3. EQUILIBRIUM AND STABILITY ANALYSIS

Any nonzero equilibrium of system (13) can be obtained by solving equation

$$\alpha - \beta X - \frac{AC}{CX + B} = 0 \tag{14}$$

Since the third term of the left hand side is a hyperbola being strictly decreasing and convex, we have the following possibilities.

Case 1. No positive equilibrium exists (Figure 1)

From equation (13) we see that $\dot{X} < 0$ for all $X > 0$. Hence for arbitrary positive initial stock $X(0)$, $X(t)$ decreases and converges to zero leading to the extinction of the fish stock. Hence the stability region for the initial fish stock is empty.

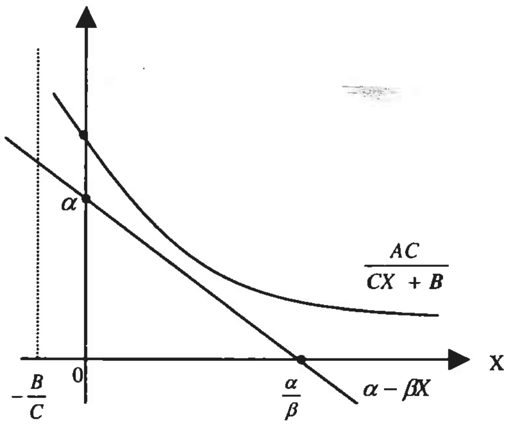

Figure 1. Case 1 with no equilibrium.

Case 2. A unique positive equilibrium exists (Figure 2)

From equation (13) we see that if $X(0) > \overline{X}$, then *X(t)* decreases and converges to $\overline{X}$. If $X(0) = \overline{X}$, then *X(t)* remains $\overline{X}$ for all future times, and if $X(0) < \overline{X}$, then *X(t)* decreases and converges to zero, resulting in the extinction of the fish stock. Hence the stability region for *X*(0) is the interval $\left[\overline{X},\infty\right]$.

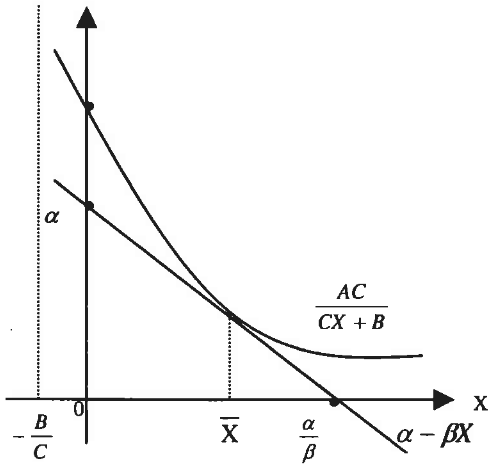

Figure 2. Case 2 with a unique positive equilibrium.

Case 3. Two positive equilibria exist (Figure 3)

From equation (13) we see that $\dot{X} > 0$ if $\overline{X}_1 < X < \overline{X}_2$; $\dot{X} < 0$ if $X < \overline{X}_1$ or $X > \overline{X}_2$; and $\dot{X} = 0$ if $X = 0$, $\overline{X}_1$ or $\overline{X}_2$. Therefore if $X(0) = 0$, $\overline{X}_1$ or $\overline{X}_2$, then *X(t)* remains constant. If $X(0) > \overline{X}_2$, then *X(t)* decreases and converges to $\overline{X}_2$; if $\overline{X}_1 < X(0) < \overline{X}_2$, then *X(t)* increases and converges to $\overline{X}_2$; and if $X(0) < \overline{X}_1$, then *X(t)* decreases and converges to zero. Hence the stability region for *X*(0) is the interval $\left[\overline{X}_1,\infty\right]$.

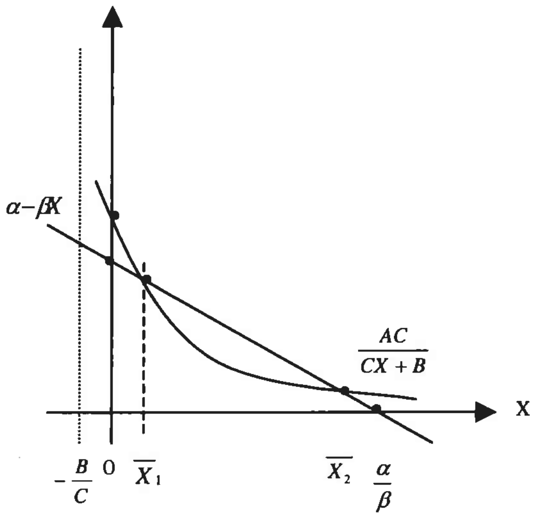

Figure 3. Case with two positive equilibria.

Case 4. A unique positive and a dummy nonpositive equilibria exist (Figure 4)

Notice that $\dot{X} < 0$ if $X > \overline{X}_2$; $\dot{X} > 0$ if $X < \overline{X}_2$; and $\dot{X} = 0$ if $X = \overline{X}_2$. Therefore by selecting an $X(0) > \overline{X}_2$, the fish stock decreases and converges to $\overline{X}_2$; if $X(0) = \overline{X}_2$, then *X(t)* remains constant, and if $X(0) < \overline{X}_2$, then *X(t)* increases and converges to $\overline{X}_2$. Hence the entire interval $(0,\infty)$ is the stability region for *X*(0).

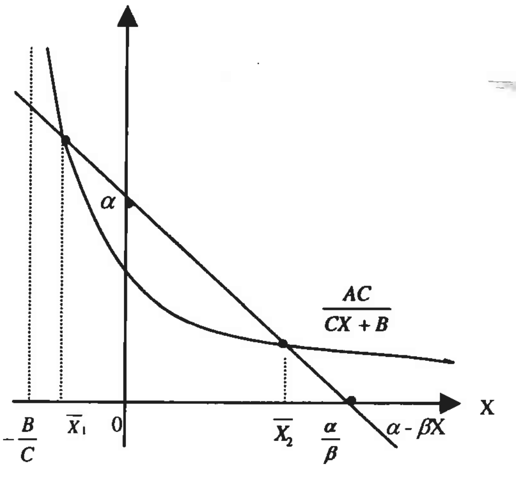

Figure 4. Case 4 with a unique positive and a dummy non-positive equilibria.

4. ENTRY INTO THE COALITION

Assume now that the n + 1[st] country joins the coalition. Before entry the harvesting rate is

$$\frac{ACX}{CX+B}$$

where A, B, and C were defined before. After entry the harvesting rate becomes

$$\frac{A'C'X}{C'X+B'},$$

where

$$A'=\sum_{j=1}^{n+1}\frac{a_j}{2b_j},\quad B'=\sum_{j=1}^{n+1}\frac{1}{b_j},\quad \text{and}\quad C'=\sum_{j=1}^{n+1}\frac{1}{\gamma_j}$$

Since all quantities A, B, and C increase as a result of the entry, no definite general comparison of the harvesting rates can be made. The harvesting rate increases as the result of the entry if and only if

$$\frac{A'C'X}{C'X+B'}\geq\frac{ACX}{CX+B} \tag{15}$$

which is equivalent to the relation

$$XCC'\frac{a_{n+1}}{2b_{n+1}}\geq AB'C-A'BC' \tag{16}$$

therefore the effect of entry on the harvesting rate depends on the actual fish stock. However, in the case of identical firms, relation (16) always holds implying that the harvesting rate increases. To show the validity of (16), let $a_1=\ldots=a_{n+1}=a$, $b_1=\ldots=b_{n+1}=b$, $\gamma_1=\ldots=\gamma_{n+1}=\gamma$, then the right hand side is always negative:

$$\frac{na}{2b}\frac{n}{\gamma}\frac{n+1}{b}-\frac{(n+1)a}{2b}\frac{n+1}{\gamma}\frac{n}{b}<0, \tag{17}$$

and the left hand side is always non-negative.

5. COMPARISON TO THE COMPETITIVE CASE

In Szidarovszky and Okuguchi (1998) an alternative model was examined in which the harvesting countries formed an oligopoly, and fishing rates were determined by the unique Cournot equilibrium. If X was the fish stock, then the total fishing rate was given as

$$S_C=\frac{2Af(X)}{1+f(X)}, \tag{18}$$

where

$$f(X)=\sum_{k=1}^{n}\frac{1}{1+2B\frac{\gamma_k}{X}}. \tag{19}$$

We have also proved that $\frac{S_C}{X}$ is strictly decreasing and convex in X, so the stability properties of the model with competition is very similar to that with cooperation. As we have seen earlier in this paper, in the case of cooperation, the fishing rate is

$$S=\frac{ACX}{CX+B}. \tag{20}$$

It is easy to see that

$$S_C>S, \tag{21}$$

that is, cooperation among the harvesting nations always reduces the harvesting rate. Simple calculation shows that inequality (21) is equivalent to the relation

$$\sum_{k=1}^{n}\frac{a_k}{1+a_k}>\frac{\sum_{k=1}^{n}a_k}{1+\sum_{k=1}^{n}a_k}, \tag{22}$$

where $a_t = \dfrac{X}{2B\gamma_t}$. This inequality is clearly true, since each term of the right hand side is smaller than the corresponding term of the left hand side.

In order to examine the long-term effect of cooperation, notice first that $\dfrac{S_c}{X}$ and $\dfrac{S}{X}$ have the common limit $\dfrac{AC}{B}$ at $X = 0$. Based on this observation, we will show that the first stock is more stable in the case of cooperation by verifying that cooperation keeps or increases the stability region for the initial fish stock $X(0)$.

Assume first that without cooperation Case 1 occurs, which is illustrated in Figure 1. Since after cooperation the value of the nonlinear curve at zero remains the same, but the curve moves downwards, Cases 1, 2, or 3 might apply to the new case. So the stability region for $X(0)$ either remains empty, or becomes the interval $\left[\overline{X},\infty\right]$ or $\left[\overline{X}_1,\infty\right]$. Assume next that without cooperation, Case 2 applies. Then after cooperation Case 3 will occur with $\overline{X}_1 < \overline{X}$, so the stability region increases from $\left[\overline{X},\infty\right]$ to $\left[\overline{X}_1,\infty\right]$. If we have Case 3 without cooperation, then after cooperation the nonlinear curve moves downwards, so we will have Case 3 again, but $\overline{X}_1$ decreases. Therefore the stability interval $\left[\overline{X}_1,\infty\right]$ becomes larger. Assume finally that without cooperation Case 4 occurs. Then after cooperation the same case will apply, so the stability region $(0, \infty)$ remains the same.

Hence in all cases the stability region remains the same, or increases, showing that cooperation among the harvesting nations makes fish stock more stable.

6. CONCLUSIONS

In this paper a dynamic model of international commercial fishing is examined, when the countries form a coalition and at each time period the profit of the coalition is maximized. The overall profit of the coalition is then distributed among the members according to cooperative game concepts. The combination of profit maximizing fishing activities with the biological growth model of fish stock results in a special dynamic system.

We have shown that there are at most two positive equilibria. The monotonic and asymptotical properties of the fish stock have been examined as a function of the initial fish stock and we have shown that the fish stock is always monotonic in time and either converges to a positive equilibrium or to zero resulting in the extinction of the fish stock. The effect of the entry of a new country is also examined. We found a sufficient and necessary condition for the harvesting rate to increase, and this condition is always satisfied in the case of identical costs and fish prices. We have also compared the harvesting rates of cooperating and competing countries, and we have presented model results to indicate that cooperation among the harvesting countries reduces the harvesting rate and makes the fish stock more stable in the long run. We finally note that the results of this paper can be used in cases when a country is divided into several regions, when the fish price of each region depends on the fish supply in that region, and each region has a regional fishing industry.

7. REFERENCES

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